HON PRECALCULUS Name: Solutions

Test Rejects From A Finely Crafted O'Brien Unit 2 Test

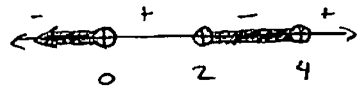
1. Solve $y^3 - 6y^2 + 8y < 0$.

(No calc)

$y(y^2 - 6y + 8) = 0$

$y(y-4)(y-2) = 0$

$y = 0, 2, 4$



$x < 0$
$2 < x < 4$

2. Solve, giving all real and imaginary roots.

(No calc)

$x^3 - 2x + 4 = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$(x+2)(x^2 - 2x + 2) = 0$

$a = 1$
$b = -2$
$c = 2$

$x = \dfrac{2 \pm \sqrt{4 - 8}}{2}$

$= \dfrac{2 \pm \sqrt{-4}}{2}$

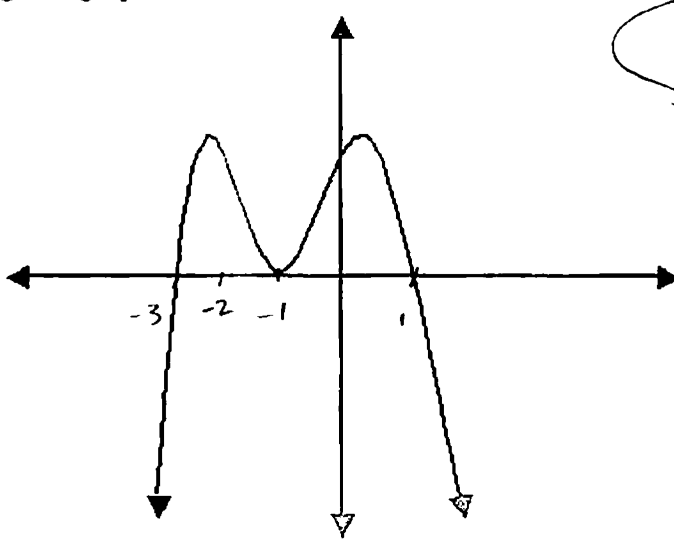
$= \dfrac{2 \pm 2i}{2}$

$= 1 \pm i$

So, $x = -2, 1 \pm i$

3. Give an equation of a polynomial function that could have the graph below. Explain why your function could have the given graph.

(No calc)



$y = -(x+3)(x+1)^2(x-1)$

Negative lead coefficient, since it opens downward

Double root to make it kiss the axis

Zeros at $x = -3, -1, 1$

4. Factor completely: $x^3 - 3x^2 - 6x + 8$

(No calc)

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$(x-1)(x^2 - 2x - 8)$

$(x-1)(x-4)(x+2)$

5.
The polynomial $x^3 + ax^2 - 3x + b$ is divisible by $(x-2)$ and has a remainder 6 when divided by $(x+1)$. Find the value of a and of b.

No calc

2	1	a	-3	b
		2	$2a+4$	$4a+2$
	1	$a+2$	$2a+1$	$4a+b+2$

$4a+b+2 = 0$

-1	1	a	-3	b
		-1	$-a+1$	$a+2$
	1	$a-1$	$-a-2$	$a+b+2$

$a+b+2 = 6$

Subtract
$$\begin{cases} 4a+b+2=0 \\ a+b+2=6 \end{cases}$$

$3a = -6$

$a = -2$

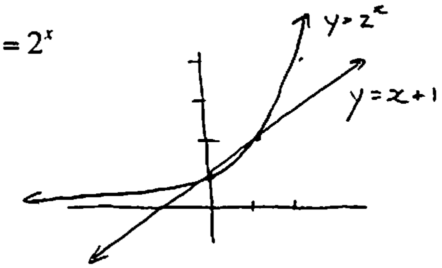
$-2+b+2=6$

$b=6$

6. Solve the following equation (Hint: consider using a graphic method).

No calc

$1 + x = 2^x$

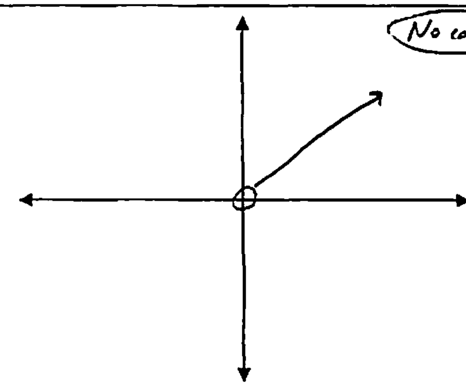


$x = 0$ or 1

7. Sketch a graph $y = 2^{\log_2 x}$. What is the domain of this function?

No calc

$y = x$, $x > 0$



8. At the right is a "solution" to the equation $100 = 18e^{4k}$.

Calc

a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$18 \cdot e^{4 \cdot 0.398} \approx 88.4$ ✗

b. Circle the error in the "solution" and give a correct solution below.

$$100 = 18e^{4k}$$
$$\ln 100 = \ln\left(18e^{4k}\right)$$
$$\ln 100 = \ln 18 \ln\left(e^{4k}\right)$$
$$\frac{\ln 100}{\ln 18} = \ln\left(e^{4k}\right)$$
$$\frac{\ln 100}{\ln 18} = 4k$$
$$\frac{\ln 100}{4 \ln 18} = k$$
$$k \approx 0.398$$

$\ln 100 = \ln 18 + \ln e^{4k}$

$\ln 100 - \ln 18 = 4k$

$\dfrac{\ln \frac{100}{18}}{4} = k$

$k \approx 0.429$

9. Given that $\log_a b \approx 1.7712$, approximate

Calc

a. $\log_a ab$

$\log_a a + \log_a b$

$= 1 + \log_a b$

$= 2.7712$

b. $\log_b a$

$= \dfrac{\log_a a}{\log_a b}$

$= \dfrac{1}{1.7712}$

≈ 0.5646

c. $\log_a \dfrac{1}{b^2}$

$= \log_a b^{-2}$

$= -2 \cdot \log_a b$

$= -2 \cdot 1.7712$

$= -3.5424$

10. Consider the expression $y = 3\ln x$. What happens to y if x triples?

No Calc

$= 3\ln(3x)$

$= 3(\ln 3 + \ln x)$

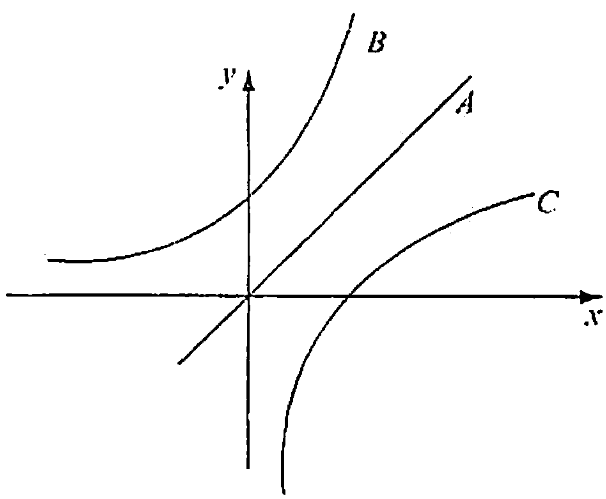
$= 3\ln 3 + 3\ln x$

Increases by $3\ln 3$.

11.

The diagram shows three graphs.

No Calc



A is part of the graph of $y = x$.

B is part of the graph of $y = 2^x$.

C is the reflection of graph B in line A.

Write down

(a) the equation of C in the form $y = f(x)$; $y = \log_2 x$

(b) the coordinates of the point where C cuts the x-axis.

$(1, 0)$

12. True or false. If false, give a reason why. No calc

a. $\log_{-2} 4 = 2$

False – base can't be negative

b. $\ln(5+x) = \ln 5 \ln x$

False e.g. If $x=1$, $\ln(6) \neq \ln 5 \cdot \ln 1$.

c. $\log_7 \frac{1}{2} = -\log_7 2$

True

d. $10^{\log 10^{10}} = 10$

False $10^{\log 10^{10}} = 10^{10}$

e. $\log x - \log 2 = \frac{\log x}{\log 2}$

False e.g. $\log 2 - \log 2 \neq \frac{\log 2}{\log 2}$

$0 \neq 1$.

f. $\log 4 = \frac{1}{2}\log 16$

True

g. $\log\left(-\frac{1}{100}\right) = -2$

False Can't take log of a negative.

h. $\log_b 8 = \log_b 8 + \log_b 0$

False Can't take log of a negative

13. Find the **exact** value of x satisfying the equation No calc

$$(3^x)(4^{2x+1}) = 6^{x+2}.$$

Give your answer in the form $\frac{\ln a}{\ln b}$ where $a, b \in \mathbb{Z}$.

$\ln 3^x \cdot 4^{2x+1} = \ln 6^{x+2}$

$x\ln 3 + (2x+1)\ln 4 = (x+2)\ln 6$

$x\ln 3 + x \cdot 2\ln 4 + \ln 4 = x\ln 6 + 2\ln 6$

$x\ln 3 + x\ln 16 - x\ln 6 = \ln 36 - \ln 4$

$x(\ln 3 + \ln 16 - \ln 6) = \ln 9$

$x = \frac{\ln 9}{\ln\left(\frac{3 \cdot 16}{6}\right)}$

$x = \frac{\ln 9}{\ln 8}$